Using Dynamic Coordinate Geometry as a Tool for Investigating Functions in a Course for Prospective Elementary School Teachers

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Abstract The pre-service teachers in this study worked individually at computers on activities that were designed to investigate aspects of functions and their properties. The activities contained introductory, sketch-making and exploratory sections. Scores on activities that involved domain and range, odd and even functions, and function composition showed evidence of first decreasing from the mean scores on earlier activities and then increasing to match the earlier average scores. A final analysis of the future teachers' overall averages on the activities showed that these measures were lower than their overall course means, and this reinforced the finding that activities involving domain and range, odd and even functions, and function composition were more challenging.

Recent research has shown that technology can enable students to: (1) make flexible connections between symbolic and graphic representations of functions; (2) exhibit greater evidence of a transition from an operational to a structural understanding of functions; and (3) perform better in creating and coordinating multiple representations of functions (Brenner, Moseley, Brar, Duran, Reed, and Webb, 1997; Clements, Sarama, and Battista, 1998; O'Callaghan, 1998; Hollar and Norwood, 1999; Kordaki and Potari, 2002). Other studies have indicated that students, including pre-service teachers, continue to have difficulty with a wide variety of tasks related to characteristics of functions and there is a need to devise learning environments that make functional relations more salient (Adams, 1997; Chinnappan, 1998). Also, from studies with prospective teachers, Cedillo (2001) and Pandiscio (2002) found that these tertiary students explored problems more deeply with mathematical software and that these students also believed that such computer applications helped them understand the ideas embedded in situations more fully.

This present study was undertaken to build on this research base by exploring the efficacy of using computer-generated dynamic representations of functions as a means for enhancing prospective teachers' understanding of characteristics and compositions of functions. The prospective teachers in the study were in the final weeks of a mathematics content course for future elementary school teachers. Topics that were taught earlier in the course included linear, probability, statistical, and spreadsheet functions, as well as statistical plots. In additional to working with spreadsheets during previous computer lab sessions, the students in the study had also had hands-on experience working with graphing calculators to investigate linear functions, statistical plots (histograms, scatter plots, and box plots) and fitting linear equations to data. As a result, the present study was also intended to use computer-generated dynamic representations of functions to deepen and extend the students' expertise with functions, especially representations, characteristics, and compositions of functions.

Prerequisites for the course in which the students were enrolled were two other quarter-long courses: one focusing on the real numbers and its subsets and the other emphasizing the geometric bases of elementary school mathematics programs. Consequently, the study also aimed to connect the students' prior work in geometry with dynamic representations of geometric functions. While enrolled in the prerequisite geometry course, some but not all of the students had gained experience with dynamic geometry software. This difference in prior experience was further exacerbated by some of the students having not completed the geometry prerequisite.

Activities

In the present study students worked individually at computers in a computer lab on Geometers Sketchpad activities designed to investigate aspects of functions and their properties. The activities were presented on worksheets that contained introductory, sketch-making, and exploratory sections, and the students had a total of five 110-minute class sessions to complete the activities. The introductory section described the activity and motivated students' interest in the activity. The sketching and exploring sections provided students with opportunities to follow construction steps and usually to answer questions about functions and their properties. During the first four class sessions students worked on two activities while during the fifth class session, students worked on one activity. Table 1 describes the activities that students worked on during each of the five class sessions.

Class Session	Activities
1	1.Circumference As a Function of Diameter 2. Segment Length As a Function of Distance in a Triangle
2	• Choosing Variables, Predicting Graphs, and Using Loci to Graph Functions 3.Introduction To Describing and Identifying Variables in Dynagraphs
3	4.Identifying, Transforming, and Comparing Graphs 5. Domain and Range in Graphs
4	6. Odd and Even Functions in Graphs 7. Evaluating Function Composition and Modeling Composed Functions with Dynagraphs
5	8. Iteration of Arithmetic Operations on Cartesian Coordinates of Points

Table 1. Descriptions of Activities during Each Class Session

Note that all activities in Table 1 are numbered except the first activity of class session 2. This activity was not numbered since it was the only one that did not require students to answer questions. Students' work on this unnumbered activity was neither scored nor analyzed for this report.

Activities during the first class session served as an introduction to the types of questions that dynamic coordinate geometry opens for study. Particularly, during the first activity of the first class session students investigated a circle's circumference as a function of its diameter. In the second activity of the first class session, students explored the length of the segment that had endpoints on two sides of a triangle and was parallel to the third side, as a function of the segment's distance from the third side. In both of these activities the students explored the relationships between measurement and graphs that they had made. Also, in both activities the underlying functional relationships were linear and of the form $f(x) = cx$ for $c = \pi$ in the first activity and c = the ratio of a length to a distance in the second activity.

Activities 3 to 7 used dynagraphs, a type of graphical representation that was developed to facilitate the process of understanding functions. The developers of dynagraphs intended that these representations would provide students with a graphical expression of the input-output view of functions (Goldenberg, Lewis, and O'Keefe, 1992). As a result dynagraphs may be viewed as an intermediate step between input-output machine models of functions and graphs of functions on a Cartesian coordinate system. In particular, dynagraphs are graphs of functions that are typically displayed on a coordinate system whose x- and y-axes are parallel to each other.

Students in this study used dynagraphs in each of the class sessions except the first and fifth. Dynagraphs were introduced in the second and third class sessions and then used as a means of investigating the domain and range of functions, odd or even functions, and function composition during class sessions 3 and 4. The first activity of class session 2 contained no questions for students to answer. Instead, students experimented with choosing independent and dependent variables from their choices of an assortment of geometric figures. Students then predicted the graphs of the resultant functions, and checked these predictions by using loci of plotted points to sketch the graphs of the functions. In the second activity of class session 2, students generally described and particularly identified the numerical inputs and outputs of dynagraphs.

The first activities of the third class session provided students with opportunities to investigate what properties of functions were more evident when their graphs were displayed on a coordinate system with x- and y-axes parallel rather than on the more common perpendicular axes of a Cartesian coordinate system. This activity involved identifying and transforming dynagraphs to Cartesian graphs and comparing dynagraphs and Cartesian graphs. Specifically, students matched dynagraphs to their symbolic representations, $y = f(x)$ and described how changing input and output indicators effected Cartesian points.

The second activity of the third class session and activities during the fourth class session opened opportunities for students to build on their initial work with dynagraphs in order to study the notions of domain, range, odd, even, and composition as they apply to functions. Put another way, an aim of the third and the fourth class sessions was to supplement the properties of functions with analogues in coordinate geometry that were so insightful that the nature of a particular functional property became readily evident.

In the second part of class session 3, students identified the domain and range of functions from their dynagraphs and their Cartesian graphs. During the first part of class session 4, students explored symmetries in Cartesian graphs and in dynagraphs to determine a function's parity (whether a function was odd, even, or neither). After identifying the parity of functions from their graphs, students were then asked to use this experience to determine parity without using the graphs. During the last part of class session 4, students evaluated particular function compositions and then used dynagraphs to model and find the values of other composed functions. Concluding questions in this part of the class session were aimed at enabling students to characterize a special feature (e.g., equality of input and output) in the composition of inverse functions.

In the fifth and last class session students used iteration of arithmetic operations on the Cartesian coordinates of points and the loci of these iterated points as a means representing functions and their graphs. These iterative activities provided an approach that aimed to use technology to bridge the symbolism of functions and the visual nature of their accompanying graphs.

While doing the activities students were encouraged to ask questions of one another or of the instructor, who was available throughout the class sessions. Students frequently asked the instructor about details of the activities and specific questions were often about how to carry out constructions or other steps specified on the worksheets, or how to phrase answers to questions on the worksheets. Students were also free to work together on worksheets during the class sessions so long as each student turned in his/her own worksheet by the end of the session.

Students' performance

A class reader evaluated each student's performance on the worksheets and the reader calculated a score for each activity. Scores were recorded as a percent: (points earned) / (total points possible) * 100 for each activity. Table 2 shows the students' percentage scores and descriptive statistics for each of the numbered activities.

	ACTIVITY ONE	ACTIVITY TWO	ACTIVITY THREE	ACTIVITY FOUR	ACTIVITY FIVE	ACTIVITY SIX	ACTIVITY SEVEN	ACTIVITY EIGHT
Maximum Possible	100	100	100	100	100	100	100	100
Student1	92.86	0	87.5	73.68	50	50	80	75
Student2	100	75	87.5	100	64.29			100
Student3	64.29	25	81.25	94.74	42.86	77.78	60	58.33
Student4	100	0	100	94.74	71.43	50	46.67	100
Student5	85.71	62.5	75			50	93.33	58.33
Student6	100	100	87.5	78.95	50	55.56	80	83.33
Student7	85.71	87.5	75	84.21	35.71	88.89	80	100
Student8	100	100	100	73.68	85.71	94.44	80	100
Student9	71.43	0	56.25	73.68	21.43	38.89	66.67	41.67
Student10	78.57	12.5	25			44.44	86.67	58.33
Student11	85.71	0		63.16	78.57	61.11	73.33	100
Student12			75	73.68	64.29	50	80	58.33
Student13	92.86	62.5	100	100	100	94.44	86.67	100
Student14	100	100	100	100	100	94.44	86.67	100
Student15	28.57	0	75	57.89	7.14	33.33	46.67	75
Student16	100	100	100	100	64.29	94.44	93.33	100
Student17	71.43	0	81.25	73.68	21.43	33.33	0	58.33
Student18	100	0	31.25	73.68	71.43	72.22	60	33.33
Mean	85.71	42.65	78.68	82.24	58.04	63.72	70.59	77.78
Std Dev	18.33	42.67	21.97	13.66	26.59	22.15	22.53	22.57
Median	92.86	25	81.25	76.32	64.29	55.56	80	79.17
Count	17	17	17	16	16	17	17	18

Table 2: Students' Percent Scores on the Eight Activities

Analysis

Notably, the highest and lowest average scores were for the first and second activities (means: 85.71 and 42.65; medians 92.86 and 25, respectively). Activities 3 and 4 introduced dynagraphs and students' average scores on these activities were close to 80%. This contrasts with the average scores on Activities 5, 6, and 7, wherein students used dynagraphs to investigate the domain and range concepts, as well as the notions of odd or even functions and function composition. The students' mean scores on these properties and applications of functions showed some evidence of improvement from the earlier to the later activities. Students' average scores on Activity 8, the iterative activity with no dynagraphs, nearly matched the averages on Activities 3 and 4, the initial dynagraph activities. Table 3 shows the overall activity means, overall class means, and the differences of these two means for the students who participated in the study.

	OVERALL ACTIVITY MEAN	OVERALL COURSE MEAN	OVERALL COURSE MEAN MINUS OVERALL ACTIVITY MEAN
Student1	63.63	69.27	5.64
Student2	87.8	73.54	-14.26
Student3	63.03	76.03	13
Student4	70.36	75.71	5.35
Student5	70.81	76	5.19
Student6	79.42	83.07	3.65
Student7	79.63	88.99	9.36
Student8	91.73	95.81	4.08
Student9	46.25	62.09	15.84
Student10	50.92	76.3	25.38
Student11	65.98	71.69	5.71
Student12	66.88	77.29	10.41
Student13	92.06	83.9	-8.16
Student14	97.64	92.12	-5.52
Student15	40.45	79.15	38.7
Student16	94.01	92.59	-1.42
Student17	42.43	77.44	35.01
Student18	55.24	81.94	26.7
Mean	69.98	79.61	9.7
Std Dev	17.78	8.46	13.85
Median	75	77.37	5.67

Table 3: Overall Activity and Class Means and Differences

Figure 1 displays two plots, a histogram and a normal-quartile plot, concerned with the students' overall course means minus their overall activity means. The histogram has an appearance that suggests the bell-curve shape of a normal distribution. The approximately linear shape of the accompanying normal-quartile plot of the mean differences reinforces this suggestion. Applying a single sample t-test for the null hypothesis of $\mu \leq 0$ to the difference data produced a p-value of .005 which indicated that the null hypothesis could be rejected in favor of the alternative that $\mu > 0$ (Figure 2). This was evidence that the students' overall course means were higher than their overall activity means. Comparable analyses for differences between students' scores on consecutive activities gave no plots that reflected distribution normality and no basis for further tests of the means.

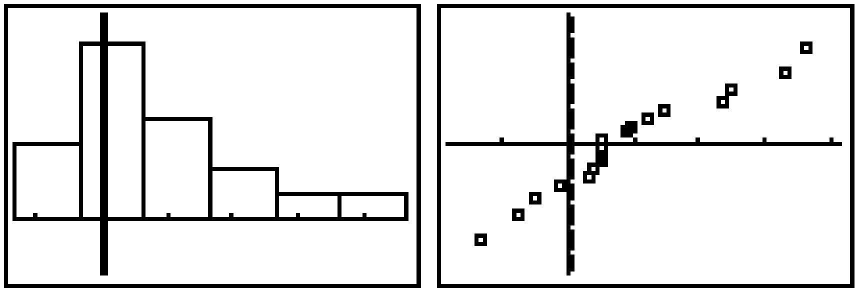

Figure 1: Histogram and Normal-Quartile Plot of Students' Overall Course Means Minus Their Overall Activity Means

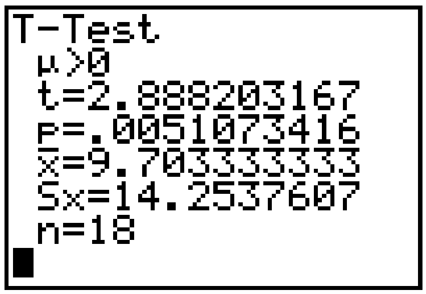

Figure 2: T-Test Results for Overall Course Means Minus Overall Activity Means

Interpretation

Activities 1 and 2 were intended to introduce or re-introduce the students to functions and the dynamic geometry software that would be used in the subsequent activities involving dynamic graphs. Students' scores on the first two activities showed considerable variations. Particularly, students' mean scores on the first Activity were the highest of all the activities while the mean scores on Activity 2 were the lowest of all the activities. This disparity may reflect the fact that only some of the students had previous experience with the software.

Activities 3-7 were the focus of the study in that they provided students with a systematic development of key aspects of functions and operations on functions. Students' mean scores on Activities 3 and 4 (78.68 and 82.24, respectively) nearly matched the students' overall class mean for the entire course (79.61). Mean scores on Activities 5, 6, and 7 were 58.04, 63.72 and 70.59, respectively. These means showed evidence of first decreasing from the mean score on Activity 4 and then gradually rising. These activities formed the heart of the explorations into the pivotal aspects of functions and their compositions, and as such might be regarded as the most intellectually challenging of the activities. Further analysis of students' overall mean on these activities showed that these means were lower than the overall course mean ($p = .03$). This result reinforced the notion that Activities 5-7 were more demanding for the students.

Collectively, Activities 3-8 yielded performance scores that exhibited notable consistency in terms of both dispersion and centrality. This finding was particularly evident when the scores on the activities that involved topics less familiar to the students (Activities 5-7) were excluded from the comparison. Further examination of the scores on Activities 5-7 showed that performance scores on Activity 5 (domain and range) and Activity 6 (odd/even functions) were considerably lower than those on Activity 7 (function composition). The instructors suggested that the unique representation of composite functions that dynagraphs provided may have served to stimulate students' interest in function composition more than was the case with domain, range, and functional parity, which were presented in relatively more traditional ways with the dynagraphs.

Activity 8 was intended to provide an alternative dynamic representation of functions as a conclusion to the sequence of activities. The mean scores on Activity 8 (77.78) showed evidence of matching students' overall mean for the entire course (79.61).

As noted previously, students who participated in this study should have completed a prerequisite course in informal geometry that included some initial work with the dynamic geometry software. The fact that some students had not completed this prerequisite may have affected student performance on the dynamic coordinate geometry activities. Particularly, student performance on Activities 1 and 2 (the introductory Sketchpad activities in this study) were likely impacted by students' prior experience with dynamic geometry software. The instructors made a special note of the fact that the seven students who scored zero points on Activity 2 appeared to have major difficulties completing the initial Sketchpad work in Activity 1 within the allotted time.

Also as noted earlier, the students should have completed a course on the real number systems and their operations, properties, and applications. One part of this number systems course deals with relations and functions, and within this part of the course there is some material on domain, range, and function composition. However, as was the case with the informal geometry course, not all students may have completed this course prior to, or even concurrent with, the course in which the present study was conducted. Moreover, some students who had completed the number systems prerequisite, may not have studied the topics of domain, range, and function composition. In either case, the fact that some students had not studied domain, range, and the composition of functions previously may have also affected student performance in this study.

The topic of the functional parity was likely a new topic and a new activity for nearly all the students in this study. Except for Activity 2, this activity (Activity 6) had the lowest median score of all the activities in the study.

Ultimately, the question of the effectiveness of these activities for enhancing students' understanding of the characteristics and compositions of functions remains an open one. Dynamic geometry software is one of the growing array of software that have the potential to enhance student understanding and to support new types of pedagogy (Conference Board of the Mathematical Sciences, 2001). Prospective teachers at all levels need experience in using computer applications in course across the undergraduate curriculum. By the same token, mathematics educators need to continue to explore how these computer tools can play a meaningful role in future mathematics curricula.

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