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Simultaneous activation of conceptual and procedural mathematical knowledge by means of ClassPad

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If we agree that the main goal of mathematics education is to develop both procedural and conceptual knowledge and to make links between the two, a very important research question regarding technology-based mathematics education is how different pedagogical solutions affect the relation between the two knowledge types. This paper highlights some examples concerning that question by utilizing recently-introduced hardware and software innovations. These allow constructions of a mental bridge between concrete and abstract objects by simple drag-and-drop activities, as from geometric to algebraic window, and vise versa.

Knowledge distinction and four relations

After a careful analysis of the studies concerning conceptual and procedural mathematical knowledge, we (Haapasalo & Kadijevich 2000) noticed that it is especially the dynamic and semantic view of conceptual knowledge, which should be highlighted more clearly. In our view, the two knowledge types can, in some cases, be distinguished only by the level of consciousness of the applied actions. We make the following distinction:

• *Conceptual knowledge* (abbreaviated to ***C*** through this paper) denotes knowledge of and a skilful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representation forms.

• *Procedural knowledge* (abbreaviated to ***P***) denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation forms. This usually requires not only the knowledge of the objects being utilized, but also the knowledge of format and syntax for the representational system(s) expressing them.

P often calls for automated and unconscious steps, whereas ***C*** typically requires conscious thinking. However, ***P*** may also be demonstrated in a reflective mode of thinking when, for example, the student skillfully combines two rules without nowing why they work.

Four relations concerning the theoretical/empirical links between ***P*** and ***C*** (the ***P-C*** links) can be realized in the existing literature on this topic (see Haapasalo & Kadijevich 2000, pp. 145-146). These are:

- *Inactivation view* (*I*): ***P*** and ***C*** are not related (Nesher 1986; Resnick & Omanson 1987).
- *Simultaneous activation* view (*SA*): ***P*** is a necessary and sufficient condition for ***C*** (Hiebert 1986, Byrnes & Wasik 1991; Haapasalo (1997).
- *Dynamic Interaction* view (*DI*): ***C*** is a necessary but not sufficient condition for ***P*** (Byrnes & Wasik 1991).
- *Genetic view* (*G*): ***P*** is a necessary but not sufficient condition for ***C*** (Kline 1980, Kitcher 1983, Vergnaud 1990, Gray & Tall 1993, Sfard 1994).

Having in mind different student abilities, various teaching approaches and topics with associated problems it is appropriate to stress that these four views do not evidence any general conclusion regarding the relation between ***P*** and ***C*** (cf. Kadijevich 2003). In this paper we highlighten some pedagogical implications of the *DI* and *SA* views.

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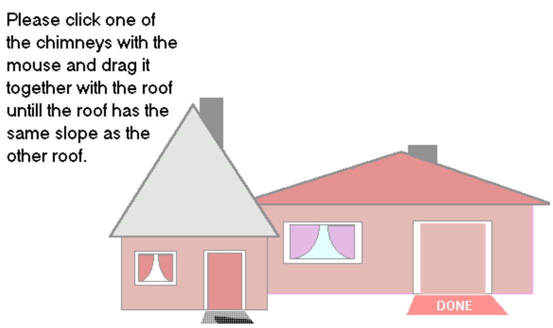
For finding pedagogical interpretations for the four views above, we define two approaches as follow:

- *Educational approach* is based on the assumtion that ***P*** depends on ***C***. Thus, the logical background is *DI* or *SA*. The term refers to educational needs, typically requiring a large body of knowledge to be transferred and understood.
- *Developmental approach* assumes that ***P*** enables ***C*** development. The logical background is *G* or *SA*, and the term reflects the philogenetic and ontogenetic nature of mathematical knowledge.

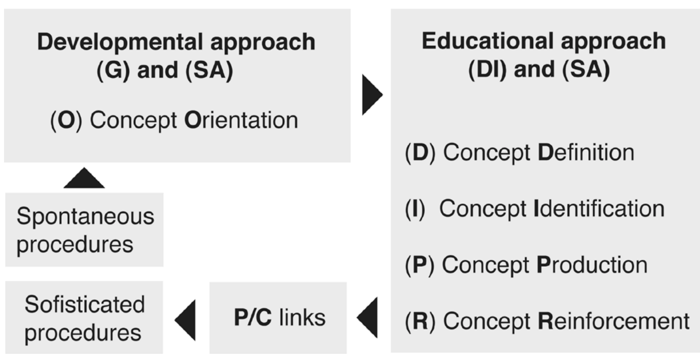
We refer to Haapasalo & Kadijevich (2000 pp. 147-153) for a detailed characterization of the two approaches, making here just the remark that the dominance of ***P*** over ***C*** seems quite natural both in the development of scientific and individual knowledge. A reasonable pedagogical idea in any topic could be to go for spontaneous ***P***. On the other hand, it seems appropriate to claim that the goal of any education should be to invest on ***C***.

Dynamic interaction and simultaneous activation

We will start representing the two views by using examples from the first MODEM study concerning the conceptual field *Proportionality - Linear Dependence - Gradient of a Straight Line through Origin*, denoted shortly by C_1 hereafter. For being able to represent how the educational approach is utilized, and how the developmental approach is used to trigger it, the framework theory[2] should be linked to the considerations (Haapasalo (1997, 2003). Having in mind our remark above, we would like to start with a spontaneous ***P***. We therefore restrict the construction space by simplifying C_1. Gradient is considered as a concrete *slope*, at first. The figure on the left illustrates a nice example of developmental starting harbor for C_1. Children can handle the situation by using spontaneous ***P*** based on every-day experiences without any explicit thinking of the mathematical relations between the objects.



It is this kind of orientation (*O*) that forms the first phase of the *concept building.* Concerning the *DI* method within other phases (*D*efinition, *I*dentification, *P*roduction and *R*einforcement) in the figure below, we refer to Haapasalo (1997, 2003). The orientation basically utilizes developmental approach: the interpretations are based on pupils' mental models and more or less naive procedural ideas. These act like a wake-up voltage in an electric circuit that triggers another, more powerful current to be amplified again. ***P*** and ***C*** start to accelerate each other, offering a nice opportunity to use *SA*, for example. Being at the intersection of the logical definitions of the two approaches, *SA* links the developmental approach and educational approach in the most natural way.



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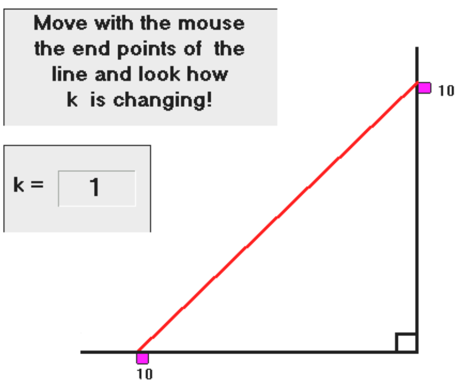
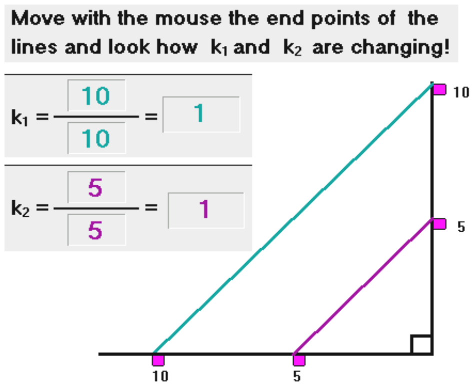
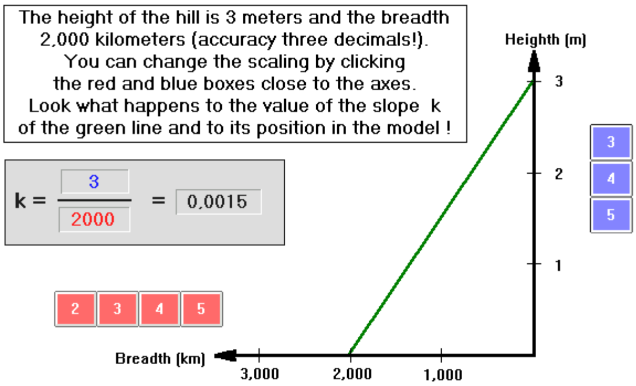
[2] The software can be downloaded at http://www.joensuu.fi/lenni/programs.html . Examples of interactive Java applets utilizing the *SA* method can be found at http://www.joensuu.fi/lenni/SA/conics.html.

The figures below represent *SA* in the following way: the pupil can manipulate the concrete slope with the mouse and look how its abstract symbolic representation is changing. On the left-hand screen (s)he has to handle just few data chunks, whereas on the right-hand screen (s)he should have some metacognitive abilities to regulate his/her own learning (students and teachers often change too many things at the same time; cf. Haapasalo 2003). Note that in *SA* method the construction does not need to begin from the concrete or abstract, but between abstract and concrete, and even between abstract things.







For moving from the concrete slope to the abstract mathematical concept *gradient* we utilize the *SA* method again. The figure on the left shows how the pupil can manipulate concrete slope (procedurally) and watch up the changes in its mathematical model (conceptualization), gradient: "If I model the same hill mathematically, the symbolic expression (gradient) is constant, but the visual model (slope) can vary arbitrary."

Utilizing *SA* method with ClassPad

For about 20 years, it has been possible to interpret symbolic representations as graphs by using computers. Paradoxically, students should learn to understand these symbolic representations first before being able to utilize computers in this conventional way. This strongly contradicts modern constructivist theories on learning, and we cannot be satisfied with this kind of one-way ticket. Having highlighted the basic featutes of the *SA* method, we would like to illustrate the same kind of activities by utilizing *ClassPad 300*, a modern pocket computer made by Casio (see http://www.classpad.org/Classpad/Casio_Classpad_300.htm).

Most *ClassPad* applications support simultaneous display of two windows, allowing to access the windows of other applications from the main application and to perform drag and drop activities (i.e. copy and paste actions), and other operations with expressions between the *Main Application* work area and the currently displayed screen (*Graph Editor, Graph, Conic Editor, Table, Sequence Editor, Geometry, 3D Graph Editor 3D Graph, Statistics, List Editor,* and *Numeric Solver*).

Let's start with an example, which shows how the properties of dynamical geometry programs have been extended to allow an interplay between algebra and geometry.

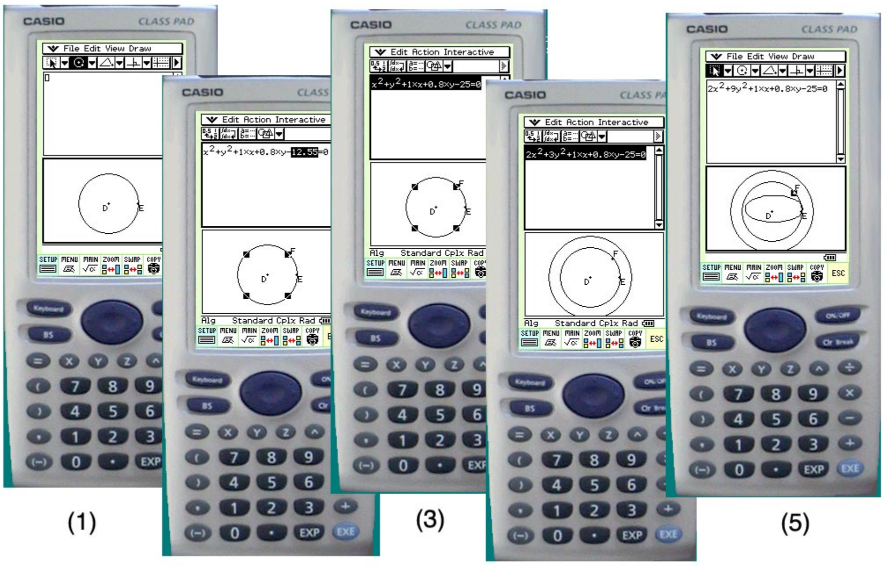
Example 1. Without knowing anything about the analytic expression of a circle, we can just play harmlessly by drawing a circle in the geometry window (1), and then drag and drop the cirlcle into the algebraic window (2). Something surprising happens: The circle seems to be expressed in algebraic form

$x^2+y^2+0.8xy-12.55=0$.

Let's manipulate (3) the equation by changing the constant to 25, then drag-and-drop it to see the new circle (4). It seems that only the radius changes. Let's go back to the algebraic window to do more manipulations (5). This time, let's change the coefficients of the second degree variables: 1 to 2 and 1 to 9:The equation

$2x^2 + 9y^2+0.8xy-12.55=0$

seems to make an ellipse.

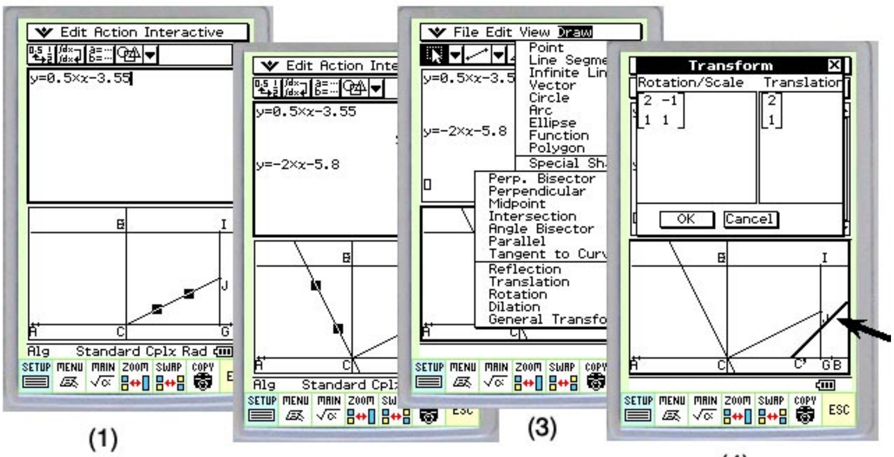


(1) (2) (3) (4) (5)

Anticipating that some readers might question this kind of *informal mathematics*, we would like to point out that the aim of the used *SA* method here has been to enhance *mental links made by the student* and not to produce any exact mathematics, yet. Of course, *ClassPad* modules would allow us to continue the above analysis on a more exact level by using plotting options as 'Sketch' or 'Conics'. The table below shows other types of expressions you can drag and drop between the 'Main Application' and the 'Geometry' window.

Main Application window:	*Geometry window:*
Linear equation in x and y	An infinite line
Equation of circle in x and y	A circle
2-dimensional vector	A point or vector
2 • 2 matrix	A transformation
Equation y = f(x)	A curve
n • 2 matrix	A polygon (each column represents a vertex)

• *Example* 2. Let us construct in the Geometry window (1) the segment CJ. A drag-and-drop activity produces its algebraic presentation 0.5x-3.55. Now we construct a line through C perpendicular to CJ, and are curious to see its equation (2). Interestingly the gradient changed from 1/2 to –2. This gives us a hypothesis, which might be worth of testing. However, this time we would like to play with 'General Transformation' (3). Two matrices appeared in the algebraic window. When filling and dragging-dropping them, the segment moved to a new place (marked by arrow). We make a hypothesis: "A transformation seem to consist of rotation and translation, both being representable by a matrix".



(1) (2) (3) (4)

Closing remarks

There is often a conflict between ***C*** and ***P*** (cf. Haapasalo 2003). We cannot make any definitive conclusions about how, even less in which order, students' knowledge develops in each situation and in each topic. Even the most abstract concepts can be based on their spontaneous ideas. This, however, does not predestine any order for the activities, because it is the pedagogical framework that matters. Our position is that *doing* should be cognitively and psychologically meaningful for the student. Building a bridge between geometry and algebra - one of the major foci in the history of mathematics - is just one opportunity to utilize *ClassPad*. Even if just imagination of the user might put limits for inventing of *SA* environments, most operations are just too complicated to be realized without obtaining first basic routines to use the equipment. That may be complicated and time-consuming with a 600-page user's guide, referring to a non-optimal user interface (cf. Carrol 1990, p. 8; Norman 1986). We still believe that *ClassPad* is a promising step towards technology that would revitalize the making of mathematics even on students' free time. A detailed analyse of TIMSS and PISA results reveal (Kupari 2003, Törnroos 2003) that it is not necessarily the school teaching that impacts on students mathematical knowledge. This makes educational research interesting - which factors in our education are important for the development of thinking abilities? If we accept the assumption that the main task of education is to promote a skilful 'drive' along knowledge networks so as to scaffold pupils to utilize their rich activities outside school, it seems appropriate to speak about an educational approach in the sense of our paper(s).

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