Some Problems of Using Maple in Teaching Mathematics

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Abstract: The article provides information on the experience in the application of computer algebra system (CAS) in teaching of mathematical analysis in teachers' training. New software products and information technologies bring quite new problems, but also new challenges especially in didactics and teachers' training. Some experience in the project oriented problems, whose solution is supported by MAPLE, are described.

Key words: Computer algebra system, mathematics teaching, examples.

Introduction

Presently the role of computer in instruction of mathematics is continuously discussed. The supporters of mathematical programs believe that the students are unnecessarily overloaded with theory and they try to reduce it. On the contrary, the opponents of the computer-aided instruction refer to a mindless application of these programs due to which the students are convinced that algorithm can be applied to everything. In fact, the mathematical programs can facilitate the student to solve the given problem, however without knowledge of respective theory the student is not able to find always a correct solution. Thus the computer-aided procedures can often result in incorrect conclusions.

The article provides short information on teachers' training in the Czech Republic at the University of South Bohemia. The instruction proceeds with the application of mathematical software, specifically in Maple program. Maple is a computer environment developed at the university in Waterloo, Canada, for an easier application of mathematics. It ranks among interactive programs that, contrary to standard programs for numerical calculations, model mathematical operations with symbolic expressions.

1* This paper is supported by grant MSM 124100006
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Using Maple in education of mathematical analysis

Using Maple in education of mathematical analysis can be divided into two stages, namely

Introduction in Maple – familiarisation with basic statements, solution to simple tasks

Solution to non-standard tasks

a) Introduction in Maple

In the first and second workshop lessons, the students are familiarised with basic statements in Maple program by means of the manual and Help menu. They improve their knowledge and skills step by step by solving given problems and experimenting. The advantages of using Maple in the lectures such as modelling, experimentation, visualization, animation, all of these performed in a simple way, are well known features. After being familiarised with the basic statements the students get simple tasks. In this stage of teaching, certain problems occur. If the set tasks require learnt procedures only, if students are familiarized with basic statements in Maple, they solve this task easily with the application of computer through the depression of several keys. Thus, after calculation of several such examples, the students justifiably believe that they would resolve other tasks even without a thorough knowledge of mathematical theorems and definitions. This can be demonstrated on the example as follows.

Example 1

Determine the surface area limited with curves set by equations

$$y = \frac{x^2}{x^2 - 4}, \quad y = x^2 - 3$$

and lying between straight lines x=2 and x=-2.

To resolve the set task without Maple application, the students need the following abilities and skills:

- Drawing a graph
- Understanding the geometrical meaning of a definite integral
- Computing a definite integral

In a CAS environment the students only need understanding of the geometrical meaning of a definite integral and knowledge of three statements, i.e.

- For representation of the area shape
- For determination of limits *a, b* of Riemann integral
- For calculation of the integral

The above example can be incorporated into the group of tasks solution to which without the computer application is on the one hand very strenuous, because it involves a lot of mechanical calculations, but on the other hand these tasks do not require a thorough knowledge of respective issue, but only learnt procedures.

b) *Solution to non-standard tasks*

In the second instruction stage the instructor focuses on the examples when a student without a thorough knowledge of theory and its understanding with the application of Maple program during solution draws erroneous or incomplete conclusions. The students frequently do not come to any conclusion, as they cannot express the given problem in the mathematical way, so there is nothing to be solved. The aim of the computer-aided instruction of mathematics is to develop the feeling of the students for estimation of the solution, their ability to resolve a non-standard task, to develop their functional thinking.

Below you can find some examples whose idea consists in the fact that during their solution the students cannot search for a solution according to learnt algorithms incorporating standardised solutions, but they have to orientate themselves in the problem and be able to model real world situations. The computer is only a guide for a more rapid receipt of results.

Example 2

Detect the first derivative of function f in all points of its domain, where

a) $$f : f(x) = |x| \cdot e^{-|x-1|},\ x \neq 0 \qquad f(0) = 2$$

$$f : f(x) = \sqrt[3]{x^3 - 6x}$$

Solution: during their solution, students disregard the points that after mechanical deriving are eliminated from the domain of a derivative function. The derivative in these points shall be determined from the definition or with the application of theorems of differential calculus. In the event of infinite derivative, students have problems with its geometric interpretation.

Representation of the graph of given function in Maple program will facilitate the students the problem.

Example 3

Prove inequality

$(\forall x > 0) \qquad x > \ln(1+x)$

$(\forall x) \qquad \left(e^x + e^{-x} \geq 2 + x^2\right)$

Solution: In a) it is sufficient to take function $f(x) = x - \ln(1+x)$ and to prove that this function is increasing $(\forall x > 0)$. Analogical solution is applied to b).

Example 4

a) The main ropes of a rope suspension bridge are fastened on the pillars with the interval of 250 metres one from another and that are suspended in the shape of parabola whose lowest point lays 50 metres from the point of suspension. Find out the length of a suspension rope.

Solution: The task shall be converted to mathematical language. Subsequently the solution leads to calculation of integral $\int_0^{125} \sqrt{1+\left(\frac{4}{625}x\right)^2}\,dx$.

In the plane there are given points A and Q different from A (their distance is indicated a). In point Q a lamp pole is to be built. What height of the pole shall be selected for the maximum illumination of point A? The illumination intensity I of point A is given by formula

$$I = c\frac{\sin\varphi}{r^2},$$

where c indicates illuminating power of the source (for us a positive constant), r indicates the distance of point A from source of light Z and φ is the size of angle, at which the light beam of source Z slants upon the surface in point A.

Solution: The task shall again be converted to mathematical language. The solution leads to search for the maximum of function $I : I(x) = c\dfrac{x}{\sqrt{\left(a^2 + x^2\right)^3}}$, where x is the pole height.

In the above examples using Maple can improve motivation for solving problems, which can be formulated in mathematical language. Students can carry out experimentation and modelling, learning by experience, by own mistakes, by own discoveries.

Conclusion

The mathematical software is a very good guide to the solution to a lot of mathematical tasks. However, it is necessary to stress that this only concerns the person who have commanded the elementals of mathematics. Without knowing the issue, the application of mathematical software often results in a lot of errors as well as an erroneous interpretation of the results. Using mathematical software and new technologies brings not only new, strong and difficult problems but also new challenges especially in didactics. Therefore, a role of teachers seems to be more challenging, similar as teachers' training. Using technologies in a proper way could lead to a better understanding of mathematical problems, to a higher creativity and thus to a more efficient education of math teachers.

The aim of the computer-aided instruction of mathematics is to develop the feeling of the students for estimation of the solution, their ability to resolve a non-standard task, to develop their functional thinking. It is necessary to focus preparation of the students on the fact that they should not search for a solution to the problems according to learnt algorithms, into which they have incorporated standardized solutions, but that they should orientate themselves in the problem, model, express their opinions and defend them. Mathematics has the best preconditions to develop such abilities of the students. However, these abilities should properly be used.

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