The MATCOS project

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Title: THE MATCOS PROJECT

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Summary

The MATCOS project was born from an analysis of the results of the National Plan for Information Science- considered by many to be disappointing- and other similar experiments, which in Italy in the 80s-90s had planned for the introduction of computer science (the computer as a means of programming) as part of Maths teaching; it reiterates the soundness of the original proposal, especially as regards the use of programming in learning Maths and underlines the need to:

- accentuate the algorithmic aspect of Maths;
- make available an accessible programming language, in the mother tongue, which is Maths orientated and suitable for the effective logical capabilities of the students.

With reference to the second point, a programming language was constructed entitled MatCos, with the aforementioned characteristics; in fact, it is in the mother tongue (Italian), it retains the classic structure of a programming language, and besides it is Maths orientated, in the sense that it uses commands linked to precise mathematical concepts.

1. Introduction

At the beginning of the 1980s, in Italy, with the diffusion of PCs even in schools, the Minister of Education set up a modernisation plan called, P.N.I.[1], for all permanent Maths teachers[2] in higher secondary schools (students 13-18) out of which came a number of new experimental teaching programmes. The most significant novelty in all this was the introduction of the use of new technology. Fortunately, apart from in certain technical institutes, computer science was not introduced as a subject in its own right, but rather as a method to be used as a learning tool in maths teaching.

It is obvious that a calculator requires the use of software, so Pascal came to be used as a programming language, rather than Basic, in schools. Between 1995 and 1997, C.I.R.D.[3] at the University of Calabria, carried out a detailed investigation in secondary schools in Calabria. The project was called " Nuovi programmi e qualità degli studi", and its aim was to evaluate the real impact of the new technology, in particular the computer in Maths teaching. The results of the research were hardly inspiring: the drive for innovation had been bogged down by the inertia of tradition[8]. Indeed, in the overwhelming majority of cases, little had changed in the traditional teaching environment, in the best cases a Pascal programme was simply set alongside a certain topic. Sporadic instances were found of the utilisation of CAS[4], in particular Derive, but even here they were restricted to the final years, who were only allowed to "see" the functions graph. There was little knowledge of Cabrì- Géomètre, and this was anyway limited to a few simple applications. Meanwhile, in schools the fashion for the use of hypertexts had taken hold, in which the students were restricted to browsing, trying to learn in a different way, but not being encouraged to think for themselves. The analysis of the results of the investigation highlighted two major lacunae:

1) The lack of training for teachers in the need to rethink methods and contents with a view to the enormous possible benefits of the introduction of the computer. In fact, the continuous and widespread use of the computer requires a re-evaluation of the importance of algorithms in the subject. One the one hand, even the official primary school programmes published in 1985[5] stated: " … *The vast research undertaken has, however, demonstrated that it is not possible to*

[1] National project for computer education.
[2] full time permanent posts.
[3] Interdepartmental centre for educational research.
[4] Computer Algebra System.
[5] Presidential decree 12 Feb. 1985, n° 104

reach the plain of mathematical abstraction without first having followed the long road that links observation to reality, the activation of mathematical thinking, problem solving, the conquest of the first levels of formalisation. The most recent teaching research, through a careful analysis of the cognitive paths which lead to the development of Maths learning, has shown great complexity, the gradualism of the growth and lines of development, that are not univocal. In this context, it is claimed that even calculating algorithms (that is, ordered procedures) and the study of geometric shapes have a far greater formative importance than the practical applications, which, at one time, justified their inclusion in the teaching curriculum."
These recommendations figure prominently in the programme of the P.N.I. and the "Brocca Commission" , where the emphasis is placed on the teaching of problems and problem solving, and the construction of algorithms is a fundamental part of such an approach. For obvious reasons, it is not possible to go into too much detail on the importance of the algorithmic and constructional aspects of this argument, but suffice it to say that certain simple aspects, which are intrinsically algorithmic, are not rendered explicit in the traditional teaching practice in Italy, and it is those aspects that this programme will address directly.
2) The objective difficulty faced by students to acquire a programming language such as Pascal or Basic, in the school Maths curriculum.
This is the starting point of the MATCOS project.

2. The MATCOS project: aims and objectives

The fulcrum is the introduction of method and constant use of the computer in the classroom for learning Maths: the students must programme, because programming develops and raises their logical capacity to construct conceptual maps and thus forces them to think and, at the same time, form generalisations. All this is one of the most vital objectives of Maths teaching; moreover, the importance of programming in teaching praxis has already been widely recognised in a number of international situations[1]. It is clear that the continual use of the computer, as a programming tool, in learning requires a highlighting of the algorithmic and, more generally, constructional aspects of Maths as a subject. It also requires a programming language, which is not only Maths orientated, but which is also accessible to students of different ages and at different stages of logical development.
The MATCOS project operates in both directions, its aims are:

- To Encourage the student to acquire an awareness of the potential of the computer by using Maths;
- To Learn Maths in an active, participatory, constructive and fruitful way by using the computer.

Within the ambit of these aims, the following specific objectives are followed:

- Consolidation of teaching of algorithmic thinking, as a first step towards the process of abstraction;
- Change the way of communicating mathematical knowledge in state schools;
- Recognise the value of the individual competence both of teachers and students in a collective dimension of learning and testing;
- in service training for teachers geared to developing their awareness of the potential of teaching-learning Maths with computers.

3. Programming language

To put the computer right at the heart of maths teaching requires an adequate programming language; thus, on the basis of the aforementioned considerations, the MATCOS project has come up with a language with the following characteristics:

1) use of mother tongue (natural); in this way students faced with the difficult task of learning new linguistic structures, which are at one and the same time artificial, rigorous and abstract, is not further burdened with the task of learning a foreign language.

2) Maths orientated, or rather with the use of mathematical terminology and particular instructions which represent specific mathematical concepts; for example, the instruction *circ (point number)* represents the circumference of centre in "*punto*" and the radius equivalent to"*number*", where these parameters have been previously defined; in general, every instruction relative to a mathematical concept has as parameters all and only those defining the concept.
3) Appropriate for the logical and mental level of precise age range, or rather modular. Each module refers to a precise age range (or school year), in such a way that the specific instructions in each module can be removed or modified in others, for example: starting from module 1.6 we find the instruction *MCD (list of numbers)* for the calculation of the Maximum Common Divider, whereas in the previous modules this is absent. Moreover, each module contains only the instructions relating to mathematical concepts suitable for the age range in question;
4) Syntax and semantics kept as simple as possible.

This language, called MatCos, contains the following:

- Classic instructions on entry/exit; assignment, cycle, conditional, arithmetic operators, Boolean;
- Specific instructions of a geometric nature (on a plain, and in Euclidean and Cartesian space) in order to work with points, straight lines, lines, angles, rotations, etc.
- Instructions of an arithmetical and analytical nature in order to work with fractions, functions, derivatives etc.
- Instructions of a statistical-probabilistic nature – in order to work with diagrams, histograms, binomial coefficients etc.

Moreover, the absence of a declaratory phase helps to reduce the bundle of syntactical rules of the usual programming language. The most important characteristic of the language is found in the instruction called "passo-passo" (step by step) which allows students to carry out every programme using one command at a time, thus visualising the intermediate results. This attribute is highly important from the didactic point of view since it gives students the chance to check each stage of the algorithm and correct any mistakes more easily. To give a better idea of the system a number of simple programmes are illustrated below.

Example 1 – Age range 11-13 years: "Maximum Common Divider with the method of successive divisions" **- MC1 -**

LISTATO PROGRAMMA	COMMENT
a = **legginum**("il più grande dei due numeri");	reading instruction for a number
b = **legginum**("secondo numero");	reading instruction for a number
q = a **Rdiv** b;	Operator remainder of division between integers a, b
esegui finquando (q<>0);	Cycle, while
a = b;	assignment instruction
b = q;	assignment instruction
q = a **Rdiv** b;	remainder of division of integers
fine;	instruction for closing cycle
stampa ("il massimo comun divisore è =", b);	printing instruction

This programme can subsequently be generalised further, by requesting an automatic comparison between a and b.

Example 2 – Age range 13-14 years: "Construction of hexagon on a circumference" - **MC2 –**

LISTATO PROGRAMMA	COMMENT
O = punto;	assignment instruction for a point
r = 80;	assignment instruction for a number
ga = circ(O,r);	assignment instruction for circumference of centre O and radius r
A = punto_su(ga);	assignment instruction for a random point on circumference
s = segmento(A,O);	instruction for construction of segment of extremes A, O
d = segmento(A,2*r,s);	instruction for construction of segment of extreme A, length 2r in direction s
D = d.Estremo(2);	instruction for referring second extreme of segment

g1=circ(A,r);	circumference centre A e radius r
B =intersezione(ga,g1);	instruction for referring the first of the intersecting points of circumferences ga e g1
F = intersezione(ga,g1);	instruction for referring the second of the intersecting points of circumferences ga e g1
g2= circ(D,r);	assignment instruction for circumference of centre D and radius r
C = intersezione(g2,ga);	instruction for referring the second of the intersecting points of circumferences g2 e ga
E =intersezione(g2,ga);	instruction for referring the second of the points g2 and ga
poligono(A,B,C,D,E,F);	instruction for drawing polygon for vertexes (A,B,C,D,E,F,)
cancella(g1);	cancelling instruction
cancella(g2);	cancelling instruction

Example 3 –Age range 13-15years: " Ruffini's rule" (elementary version) - **MC3 -**

LISTATO PROGRAMMA	COMMENT
z = legginum("valore in cui si vuole il polinomio");	reading instruction for a number
n = legginum("grado del polinomio");	reading instruction for a number
I = n;	
b =0;	
esegui finquando (I≥0);	Cycle, while
a = legginum("coefficiente del termine di I grado");	reading instruction for a number
b = b*z+a;	
I =I-1;	
fine;	instruction for closing a cycle
stampa("il valore del polinomio per x=z è," ", b);	printing instruction

This programme can be generalised for a higher scholastic level with the instruction "vector n".

Example 4 – Age range 16-18 years: "Graph of a function with calculus and relative graph of the derivative – **MC4 –**

LISTATO PROGRAMMA	COMMENT
Rifcart;	Instruction for drawing the Cartesian Reference
f = leggifunz("introdurre la funzione da editor");	Entry instruction for a function f
graficofunz(f);	Instruction for drawing graph of function f
colore(10);	Instruction for changing colour
g = derivatafunz(f);	Instruction for the calculation of analytical derivative of function f
stampafunz(g);	Printing instruction for a function
graficofunz(g);	Instructions for drawing graph of function f

4. Experimentation and evaluation

The project proceeds in three stage[9]:

- I Stage – Lower secondary school;
- II Stage – First two years Higher Secondary;
- III Stage – Final three years Higher Secondary school.

Each stage takes place in annual phases, each annual period refers to a precise school year and involves training courses for the teachers who will carry out the experimentation, and tests for the students. During the 1999-2001 three year period, a first cycle involving lower secondary schools (age range 10 – 13 years) was completed. As regards the didactic activities put forward, the reader can refer to [3] and to a synthesis to be found in [7]. The tests administered and the results can be found in [6]. During 2000-2001 we also completed the cycle relating to the first two years of (higher) secondary school. (14-15 year olds). For this activity as well, the didactic proposals can be found in [3]. Currently, the experimental phase for Stage 3 is on-going, concerning the final three year period of secondary school (16-18 year olds). Training courses for the teachers involve both a revisiting of the traditional Maths syllabus in order to highlight the algorithmic aspects and the programming in MatCos. Some examples can be found in [4] and [5].

The model used for testing learning achievement is more of a strategic- constructive nature than a training one. It is based on observation tools, which are used to monitor the activities implicit in the processes of innovation, and the effects of the introduction of this new language on the educational environment. The model consists of two tests carried out during the school year. There are ten questions for each test and are of two different types, both are graded in terms of difficulty:

- type 1 – this type requires the programme list, a work more of quality than of quantity;

- type 2 – once assigned the programme list students are asked to provide a step-by-step interpretation.

On the completion of type 1, type 2 involves much more than mere training on programming language; rather the aim of type 2 is to guide the student towards the discovery of mathematical thought. Type 1 enables the student to analyse, operate, construct, simplify, design and invent; type 2, on the other hand, helps the student recognise, interpret, analyse and synthesise. Both question types were carefully planned to have, as a starting point, the problem, and not the formula because learning does not occur through the passive repetition of actions, but gradually in stages as the student increases awareness. Both types contain learning elements, and make explicit processes which usually remain hidden. Their function is to highlight, with particular attention, logical reasoning[2]. The evaluation of learning progress recorded from a sample of students who participated in the experimentation, is taken from the analysis of the test administered during the school year. The analysis is carried through systematic quantitative monitoring and is as follows:

- each test is comprised of n°10 questions which are evaluated individually by the teacher/researcher through items[6], found at the foot of the questions themselves;
- each test is made up of two sheets

On the first sheet the teacher/researcher records the results obtained by each student on the individual questions; the second summarising sheet, on the other hand, provides an evaluation of the progress of the class as a whole. Subsequently, the results collated on the two sheets are collected and monitored by C.IR.D. with the aim of finding an overall statistical pattern of learning progress from the sample of students. Considering the specific aim of the project, the statistical investigation develops in two directions: a) number of questions b) recorded correct answers. In C.IR.D. the results obtained from the sample of students are analysed, then synoptic tables are designed relating to the results of the two tests, always referring to the various items. For a more in depth reading of the results on the various stages of the MATCOS project, the reader is invited to log on to website http://cird.unical.it , or see the bibliography[7]. From the results of this research project, an effective verification has emerged on what actually happens when one tries to learn, or one tries to teach, through the use of the computer. The use of computers accompanied by suitable learning material, linked in an appropriate manner to the classroom situation encourages the achievement of educational objectives. MatCos language has been shown to be an effective educational tool, because it allowed students to follow learning paths which respected the individual rhythms, and was differentiated to enable each student to assimilate information at his or her own pace. Through the programme, students can unpack complex concepts to arrive at simple concepts, can think up new strategies to solve problems and compare them with old ones; this gives rise, therefore to the development of mental processes and the consolidation of new forms of constructive thought.

The use of computers in the classroom has aroused considerable interest from students and it has stimulated intuitive, logical and imaginative skills. Geometric algorithms, in particular, have proved of great value, since, thanks to the specific instructions of the language, they were easy to implement. These activities have stimulated the creativity of the students involved, some of whom simulated prototypes of objects of sensible reality through recourse to mathematical concepts (segments, angles, circumferences, isomers, etc.)

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[6] insufficient, sufficient, good, no reply given.

[7] the written reports relating to the various years of the project can be obtained in printed form from C.I.R.D.

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